COMPARISON OF THE HEAT-EXCHANGE INTENSITY UNDER NONSTATIONARY AND STATIONARY CONDITIONS

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It is established experimentally that the coefficients of heat exchange in nonstationary processes differ from the coefficients of heat exchange obtained from the classical heat-transfer formulas. Experimental results are obtained in conformity with which the coefficient of heat exchange depends on the volume specific heat and geometric size of the body.

The measurement methods, description of the apparatus, and measurement results of heat fluxes and heat-exchange coefficients in the heating of metal spheres are presented in [1]. The nonstationarity of the heat-exchange process is due to the time change in the body surface temperature. The use of hollow spheres from different materials with different wall thickness but identical outer diameter permitted different rates of change of the surface temperature to be obtained.

The same experimental apparatus as in [1] was used to investigate the stationary heat-exchange process. A unit permitting internal water cooling was inserted in a hollow copper sphere with 15 mm wall thickness. The unit consisted of two steel tubes. The outer tube of 16 mm diameter and 0.5 mm wall thickness was the sphere support and fastened to it exactly as in the nonstationary tests. Water was delivered by means of an 8 mm diameter inner tube and was carried away in the gap between the tubes. There were holes on the portion of the inner tube located at the center of the sphere (the exit hole of the tube was sealed) to assure uniform cooling of the sphere. The water discharge was determined by the quantity of water which passed through the sphere in a known time interval. Heating of the water was measured by a differential thermocouple. Moreover, the temperature of the sphere wall was measured at several points. For this a PP-0.05 potentiometer was used. The test was conducted as follows. The sphere was submerged in a thermostat with water heated to 97°C; a specific discharge of cooling water was set up, and values of the temperature were recorded after the stationary state had been reached. The water discharge varied between 0 and 75 g/sec, which corresponded to a change in surface temperature from 97 to 30°C.

The magnitude of the heat flux was determined by two methods: by the discharge and heating of the water

$$q = Cp_{\text{в}} \Delta t G_{\text{в}} | F$$

and by the temperatures measured at the sphere wall

$$q = \frac{\lambda r_1 r_3}{(r_3 - r_1) r_2^2} \left\{ \frac{(r_2 - r_1)}{r_1 [r_3 (r_2 - r_1) - r_4 (r_2 - r_1)]} [(r_3 - r_1) r_4 t_4 - (r_4 - r_1) r_3 t_3] - t_3 \right\},$$

where r_3 and r_4 are the distances between the center of the sphere to the points of embedding of the thermocouple junctions, and t_3 and t_4 are readings of the corresponding thermocouples.

The temperature on the surface was determined by using the values found for the heat flux. Therefore, a dependence of the heat flux on the surface temperature was found, which is linear under the given conditions. The magnitudes of the heat fluxes determined by the two above-mentioned methods differ by not more than 2-5%. The heat-exchange coefficient found from the data obtained turned out to be 1610 $W/m^2 \cdot deg$. This value α is close to the value of the heat-exchange coefficient at the end of the process of heating the spheres [1] when the quasistationary mode sets in. This fact additionally verifies the adequate accuracy of the measurement methods used.

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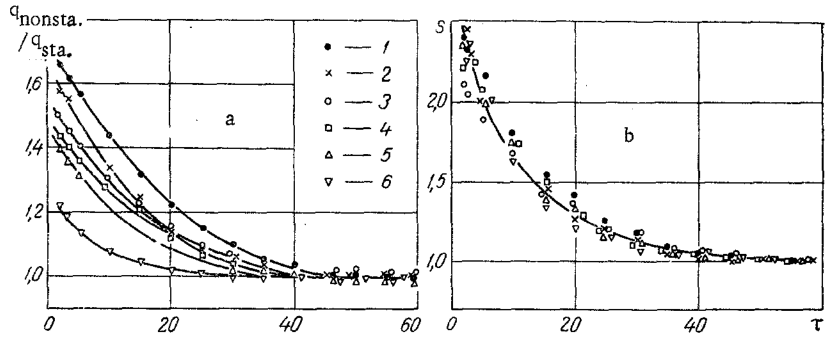

Fig. 1. Dependence of the ratio between heat fluxes under nonstationary and corresponding stationary conditions $q_{nonsta.}/q_{sta.}$ on the time τ (in sec): a) for spheres of different materials with different wall thickness; b) generalized dependence for all spheres. $S = q_{nonsta.}/q_{sta.} + (0.9 + \delta/d)(c\rho/c\rho_m)e^{-0.1\tau}$; solid curve – from (1); 1) $\delta = 0.005$ m (copper); 2) 0.015 m (copper); 3) 0.025 m (aluminum); 4) 0.025 m (brass); 5) 0.025 m (copper); 6) 0.050 m (copper).

A comparison between the Nusselt criteria and the heat fluxes under nonstationary and corresponding stationary conditions is of interest. Since it is necessary to compare the heat fluxes and heat-exchange coefficients at the same temperature difference $t_f - t_s$, it is then possible to write $q_{nonsta.}/q_{sta.} = \alpha_{nonsta.}/\alpha_{sta.} = Nu_{nonsta.}/Nu_{sta.}$. Processing results in the form of the dependence $q_{nonsta.}/q_{sta.} = f(\tau)$ showed that stratification of the curves in the parameters δ and $c\rho$ (Fig. 1a) is analogous to the dependence $\alpha_{nonsta.} = f(\tau)$. After insertion of a set of quantities taking account of the thermophysical properties of the sphere material and the wall thickness, all the curves were combined successfully into one (Fig. 1b). A function approximating the dependence obtained was found by using least squares. An expression of the form

$$y = 1 + ax \exp(-cx).$$

can be used as the initial formula for the dependence presented in Fig. 1b. Then in conformity with the method of least squares, a system of equations is formed from which the coefficients a and c are found. The final formula for the ratio between the heat fluxes is

$$\frac{q_{nonsta.}}{q_{sta.}} = 1 + 0.885\left(\frac{\tau}{\tau_*}\right)^{-0.2}\exp\left(-2.4\frac{\tau}{\tau_*}\right) - \left(0.9 + \frac{\delta}{d}\right)\frac{c\rho}{c\rho_m}\exp\left(-4.0\frac{\tau}{\tau_*}\right), \tag{1}$$

where τ_* is an interval during which the complex S (see Fig. 1b) equals 1.05 (τ_* −40 sec). The maximum relative error ξ of (1) is 5.6%.

The experimental results obtained for comparison with results in the literature can also be processes in the form of the dependence $Nu_{nonsta.}/Nu_{sta.} = f(K_T)$, where the parameter K_T characterizes the time rate of change of the relative surface temperature [2]:

$$K_T = \frac{\partial t_s}{\partial \tau}\frac{d^2}{[t_f - t_s]a_f}. \tag{2}$$

It can be noted that in conformity with similarity theory the parameter K_T is not a governing criterion since it includes dependent variables such as the surface temperature and its derivative.

For our case the coefficient of temperature conductivity of the medium is $a_f = 0.558 \cdot 10^{-3}$ m^3/h.

The sphere surface temperature is described by the equation $t_s = A_2[1 - \exp(-K_2\tau)]$. Hence

$$K_T = 64.5 \cdot 10^3 \frac{A_2 K_2 \exp(-K_2\tau)}{t_f - t_s}$$

(the coefficient $64.5 \cdot 10^3$ has the dimensionality seconds).

The dependence of $Nu_{nonsta.}/Nu_{sta.}$ on the parameter K_T (Fig. 2a) was stratified into separate curves. After insertion of the correction t_f/t_s, the stratification was successfully diminished, and the generalized dependence, presented in Fig. 2b, was obtained. It should be noted that the point spread in

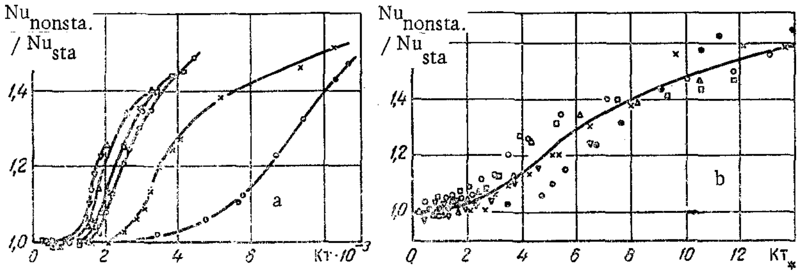

Fig. 2. Dependences of the ratio between Nusselt criteria under nonstationary and corresponding stationary conditions $Nu_{nonsta.}/Nu_{sta.}$ on the parameters K_T and K_{T*}: a) for spheres of different materials with different wall thickness; b) generalized dependence for all spheres. Solid curve) According to (3). See Fig. 1 for the notation of the points.

processing these variables is greater than in the case of using the quantities $q_{nonsta.}/q_{sta.}$, $\delta c\rho$, and τ (Fig. 1b) to generalize the test results. The dependence obtained can be approximated by a parabola. We hence have

$$Nu_{nonsta.}/Nu_{sta.} = 0.16\, K_{T*}^{0.24}, \tag{3}$$

where

$$K_{T*} = K_T \frac{t_f}{t_s} \cdot 10^3.$$

The maximum relative error in (3) is 12.5%, where K_T varies between the limits 2-14.

For $K_{T_*} = 0\text{-}2.0$

$$Nu_{nonsta.}/Nu_{sta.} = 1.0. \tag{3a}$$

It follows from Fig. 2a and b and (3) that starting with some value of the parameter, the difference in the heat-exchange intensities under nonstationary and stationary conditions will increase as K_T increases. Then the growth in the ratio $Nu_{nonsta.}/Nu_{sta.}$ slows down as K_T increases. The regularities established agree with the results in [2] obtained for the case of heat exchange during gas flow in a tube.

Using the experimental dependences (Figs. 1a and 2a), the boundaries separating the domains of the quasistationary and stationary modes can be established. If it is considered that the nonstationary effect becomes noticeable for $q_{nonsta.}/q_{sta.}$ or $Nu_{nonsta.}/Nu_{sta.} > 1.05$, then it follows from Fig. 1a

$$\tau_b = 37.0 - 0.15\, \delta c\rho, \tag{4}$$

where τ_b is the time interval in seconds during which the quasistationary heat-exchange mode is established.

The results presented in Fig. 2a permit finding a generalized dependence for the quantity K_{T_b} satisfying the following conditions: a quasi-stationary mode holds for $K_T > K_{T_b}$ and a nonstationary mode for $K_T > K_{T_b}$. This dependence is

$$K_{T_b} = 19.3 \cdot 10^3 (\delta c\rho)^{-0.52}. \tag{5}$$

The parameter K_T defined by the time derivative of the surface temperature in conformity with (2), depended on the wall thickness, properties of the material, and time in our case. Hence, it turned out to be possible to express the generalized dependence of the heat-exchange intensities by two methods: as a function of $c\rho$ and δ and as a function of K_T (formulas (1) and (3)). As has been remarked above, the left sides in the mentioned formulas are equal. There hence results that the parameter K_T is uniquely determined by the quantities δ, $c\rho$, and τ.

It should be emphasized that the parameters $c\rho$ and δ exert influence on the nonstationary coefficient of heat exchange by determining the rate of change of the surface temperature. In such an approach to this question, the contradictions between the main deductions in [2, 3], which the author indicated in [4], become rather formal.

It can be noted also that the dependence of $Nu_{nonsta.}/Nu_{sta.}$ or $q_{nonsta.}/q_{sta.}$ on $c\rho$, δ, and τ is the more convenient of the formulas in [1] and [3] for utilization, since the values of the surface temperature and its derivatives, which enter into K_T, are usually unknown in advance.

The advantage of (3) is its more general character since it connects the ratio between the heat-exchange intensities with the temperature characteristics of the surface independently of how their change is caused.

The regularities established herein can be explained by using the following model of the mechanism of the nonstationary heat-exchange process.

Because the coefficients of temperature conductivity of metals are three orders of magnitude greater than the coefficient of temperature conductivity of water, the temperature distributions near the interface between these media will be rebuilt at essentially different rates.

During heating of the sphere in the thermostat with water, the nonstationary temperature distribution in the water at the sphere surface at each instant will have a larger slope (the profile will be fuller) than the quasistationary distribution for the same values of the temperature at the wall and fluid far from the surface. The heat flux and coefficient of heat exchange in this case will be greater than in the case of a stationary temperature distribution, as follows from the Fourier and Newton laws, respectively.

Physically, for the conditions mentioned it is completely admissible that if the process of changing the surface temperature were stopped successfully, the rebuilding of the temperature distribution in the fluid would still continue for a certain time. It is due to cooling of a fluid layer near the wall until a state is built up which corresponds to the stationary heat-exchange process. This delay would evidently be greater, the greater the difference in the temperature conductivity coefficients of the media. Hence, it is more difficult to detect the effect under consideration in the case of heat exchange between metal bodies and air, whose temperature conductivity coefficients differ by just one order. This can explain the quasistationary character of the heat exchange established from test in [5]. A noticeable effect in this case will occur for a sufficient time rate of change of the surface temperature, as follows from the mechanism of the process considered above. A deviation of the intensity of the heat exchange of an air flow with a metallic body from the quasistationary value, which reached 60%, was detected in [2] for a 200 deg/sec rate of change in the surface temperature.

The dependences obtained herein of the nonstationary coefficient of heat exchange on $c\rho$, δ, τ can also be explained by starting from the mechanism of nonstationary heat exchange. If the time dependences of the surface temperatures of two spheres of identical materials but with different wall thickness or from different materials but with the identical wall thickness are compared, then the time rates of change of the temperature for the same time will be different, which will cause a delay in the temperature distribution in the fluid of a different degree. In its turn, this results in different values of the heat flux and the heat-exchange coefficient in the cases considered, where an increase in δ and $c\rho$ results in retardation of the growth of the surface temperature in the initial period of time [1], and a diminution in $q_{\text{nonsta.}}/q_{\text{sta.}}$ or $\text{Nu}_{\text{nonsta.}}/\text{Nu}_{\text{sta.}}$. This qualitative deduction agrees with the quantitative results obtained (Fig. 1a). An analogous discussion results in the deduction which agrees with test data (Fig. 1a), that the quantity $\text{Nu}_{\text{nonsta.}}/\text{Nu}_{\text{sta.}}$ should diminish, and tend to one, as time grows. Therefore, a time corresponding to a definite rate of change in the surface temperature should set in during heating of the body, after which the heat-exchange coefficient remains constant, equal to its quasistationary value [1]. The time interval to this time depends on $c\rho$ and δ (formulas (4) and (5)).

The coefficient of heat exchange in the case of a quasistationary heating process is evidently practically equal to the coefficient of heat exchange under corresponding stationary conditions.

The dependences (1), (3), (4), (5) obtained herein can be used to compute the nonstationary heat-exchange intensity of spheres with a liquid ($\text{Pr} = 1.8$) as δ/R changes from 0.1 to 1.0, the coefficient of temperature conductivity from 0.12 to 0.4 m^2/h, K_T from 0 to $9.0 \cdot 10^3$.

The effective values of the criterion Re and the velocity of fluid flow were estimated by using a dependence obtained by Kramer for spheres around which water flows under stationary conditions [6]:

$$\text{Nu}_{\text{sta.}} = 2.0 + 1.3\text{Pr}^{0.15} + 0.66\text{Pr}^{0.31} \cdot \text{Re}^{0.5}$$

The criterion is $\text{Re} = 9.8 \cdot 10^4$ and the velocity equals 0.3 m/sec. According to the results in [2], the ratio $\text{Nu}_{\text{nonsta.}}/\text{Nu}_{\text{sta.}}$ is independent of the number Re. This permits the consideration that the dependences (1) and (3) are valid not only for the number $\text{Re} \cong 10^5$ but also in at least some band of numbers Re. Further investigations of the dependence of this ratio on the criterion Re are necessary. Formulas (1) and (3) can be used to estimate the intensity of nonstationary heat exchange between metal bodies of different shapes and a fluid.

NOTATION

t, t_s	are the wall and sphere surface temperature, respectively, °C;
t_f	is the fluid temperature, °C;
τ	is the time;
r_1, r_2	are the inner and outer radius of the sphere;
d	is the sphere diameter, m;
δ	is the sphere wall thickness, m;
F	is the sphere area, m^2;
a	is the coefficient of heat conductivity of the sphere material, $W/m \cdot deg$;
q	is the specific heat flux, W/m^2;
Cp_B	is the specific heat of the fluid, $kJ/kg \cdot deg$;
G_B	is the water discharge, g/sec;
α	is the heat-exchange coefficient, $W/m^2 \cdot deg$;
a_f	is the coefficient of temperature conductivity of the fluid, m^2/h;
$c\rho$	is the volume heat capacity of the sample material, $kJ/kg \cdot deg$;
$c\rho_m$	is the volume heat capacity of copper, $kJ/kg \cdot deg$
Nu, Re	are the dimensionless numbers.

Subscript

b denotes the boundary.

LITERATURE CITED

1. A. V. Lykov, B. M. Smol'skii, and L. A. Sergeeva, Inzh.-Fiz. Zh., 18, No. 1 (1970).
2. B. M. Galitseiskii, G. A. Dreitser, V. G. Izosimov, E. K. Kalinin, and V. K. Koshkin, Izv. Akad. Nauk BSSR, Seriya Fiziko-Tekh. Nauk, No. 2, 65-76 (1967).
3. E. V. Kudryavtsev, K. N. Chakalev, and N. V. Shumakov, Nonstationary Heat Exchange [in Russian], Izd. AN SSSR (1961).
4. E. K. Kalinin, Izv. Akad. Nauk BSSR, Seriya Fiziko-Tekh. Nauk, No. 2, 77-86 (1967).
5. I. Ya. Bitsyutko, V. K. Shchitnikov, G. V. Sadovnikov, and L. A. Sergeeva, Investigation of Nonstationary Heat and Mass Exchange [in Russian], Nauka i Tekhnika, Minsk (1966).
6. W. C. MacAdams, Heat Transfer [Russian translation], Metallurgizdat (1961).